Turbulence Closure Modeling of the Dilute Gas-Particle Axisymmetric Jet

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INTRODUCTION

The turbulent flow of two-phase gas-particle suspensions occurs frequently in many processing applications. The presence of the particles, even if their volume fraction is minute, can have a pronounced effect on the structure of the underlying turbulent gas flow field. Since the motion of the gas is strongly coupled to the motion of the particles, the two fields must be considered simultaneously when trying to predict the turbulent dispersion of the particles. Basically two approaches have been employed to model these flows: Lagrangian and Eulerian. In the Lagrangian formulation a large number of particle trajectories are calculated using a previously computed fluid velocity field. Typically, the fluid field calculations do not account for the presence of the particles. Thus the two-way coupling between the phases described above is not accounted for. Recent applications using this approach are those of Shuen et al. (1983) and Weber et al. (1984). A recent Lagrangian approach that does take account of the particles' effect on the fluid turbulence is that of Gosman and Ioannides (1981). In the Eulerian approach the two phases are considered to be separate interpenetrating continua, and separate (but coupled) equations of motion are solved for each phase. The gas flow field can be described using turbulence closure models developed for single-phase flows, but these equations must be modified to account for the presence of the particles. Recent models of this type are those of Danon et al. (1977), Genchev and Karpuzov (1980), Gavin et al. (1983), Elghobashi and Abou-Arab (1983), and Pourahmadi and Humphrey (1983).

We have been developing an Eulerian model for calculating turbulent gas-particle flow (Chen and Wood, 1984a,b); in this paper we describe the application of this model to a two-phase jet in its initial region. In our previous work we have assumed that the mean velocity of the particulate phase is equal to that of the fluid phase. Using this assumption makes a quantitative description of the interaction between the two phases unnecessary. However, while this assumption is true for very small particles along the jet centerline, for larger particles (in the size range 10–100 μm), because of their inertia, there is generally a slip between the phases even on the largest scales of motion. This disequilibrium becomes more pronounced as the particle size or initial particle loading increases (cf. Girshovich et al., 1982). The main purpose of this paper will be to remove the assumption of equal mean velocities for the two phases and to use the model to compute the round jet in its initial region. These calculations will be compared with the data of Modarrass et al. (1984) and Girshovich et al. (1982), as well as with the calculations obtained using the model of Chen and Wood (1984b).

NUMERICAL MODEL

The present two-phase flow model is based on the "dusty gas" equations of Marble (1963). These equations and the turbulence model that results from manipulating them are subject to the following assumptions, which are discussed in detail in Chen and Wood (1984b):

1. The particulate phase is dilute ($\phi \ll 1$) and is made up of particles or droplets, spherical in shape and uniform in size. The particle density is large compared to the gas; $\rho_s \gg \rho$, and the model is valid when $\rho_p \sim 0(\rho)$, where $\rho_p = \rho_s\phi$ is the mean "density" of the particulate phase.
2. Both the particulate and fluid phases behave macroscopically as continua. The fluid phase is Newtonian and both have constant physical properties.
3. The mean flow is steady, axisymmetric, incompressible, and isothermal. Molecular diffusion, Brownian motion, and gravity effects on the particulate phase are negligible compared with turbulent diffusion.
4. Triple correlations involving fluctuations in the particulate phase density are negligible.
5. The interaction force term, F_{pi}, which occurs in the momentum equations of each phase with opposite sign, is given by Stokes drag law:

$$F_{pi} = \rho_p \frac{(V_i - U_i)}{t_*} \tag{1}$$

Here V_i and U_i are the velocities of the particulate and fluid phases and $t_* = (d_p^2\rho_s/18\mu)$, is the characteristic response time scale of the particles to changes in the fluid motion.

The equations describing the continuity and momentum of each phase, derived by Marble (1963), are averaged and, after applying the high Reynolds number assumptions, become

$$\frac{\partial x_2 U_1}{\partial x_1} + \frac{\partial x_2 U_2}{\partial x_2} = 0 \tag{2}$$

$$\frac{\partial}{\partial x_1}\rho_p V_1 + \frac{1}{x_2}\frac{\partial}{\partial x_2}(x_2\rho_p V_2) = -\frac{1}{x_2}\frac{\partial}{\partial x_2}(x_2\overline{\rho_p' v_2'}) \tag{3}$$

$$U_1\frac{\partial U_1}{\partial x_1} + U_2\frac{\partial U_1}{\partial x_2} = -\frac{1}{x_2}\frac{\partial}{\partial x_2}(x_2\overline{u_1'u_2'}) + \frac{\rho_p}{\rho}\left(\frac{(V_i - U_i)}{t_*}\right) \tag{4}$$

$$\rho_p V_1\frac{\partial V_1}{\partial x_1} + \rho_p V_2\frac{\partial V_1}{\partial X_2} = -\overline{\rho_p' v_2'}\frac{\partial V_1}{\partial x_2} - \frac{1}{x_2}\frac{\partial}{\partial x_2}(x_2\rho_p\overline{v_1'v_2'}) - \rho_p\frac{(V_i - U_i)}{t_*} \tag{5}$$

CLOSURE OF THE MEAN EQUATIONS

Because of the averaging process, several new variables appear in Eqs. 2–5 when compared with their instantaneous counterparts. These second-order correlations $\overline{u_1'u_2'}$, $\overline{v_1'v_2'}$, and $\overline{\rho_p'v_2'}$ represent the turbulent fluxes of momentum in the two phases and the mass flux of particles. They are modeled using a gradient hypothesis

$$-\overline{u_1'u_2'} = \nu_t \frac{\partial U_1}{\partial x_2} \tag{6}$$

$$-\overline{v_1'v_2'} = \nu_p \frac{\partial V_1}{\partial x_2} \tag{7}$$

$$-\overline{v_2'\rho_p'} = D_t \frac{\partial \rho_p}{\partial x_2} \tag{8}$$

The turbulent diffusivities ν_t, ν_p and D_t are determined from the properties of the underlying turbulent motion. In particular we use a (k-ϵ) model to determine ν_t from

$$\nu_t = 0.09 \frac{k^2}{\epsilon} \tag{9}$$

where the kinetic energy per unit mass of the fluid turbulence, k, and its dissipation rate are given by

$$U_1 \frac{\partial k}{\partial x_1} + U_2 \frac{\partial k}{\partial x_2} = \frac{1}{x_2}\frac{\partial}{\partial x_2}\left(x_2 \nu_t \frac{\partial k}{\partial x_2}\right) - \overline{u_1'u_2'}\frac{\partial U_1}{\partial x_2} - \frac{k}{3}\frac{\partial U_1}{\partial x_1} - \epsilon - \underline{\frac{2k}{t_*}\frac{\rho_p}{\rho}[1 - e^{-0.5t_*\epsilon/k}]} \tag{10}$$

$$U_1 \frac{\partial \epsilon}{\partial x_1} + U_2 \frac{\partial \epsilon}{\partial x_2} = \frac{1}{x_2}\frac{\partial}{\partial x_2}\left(x_2 \nu_t \frac{\partial \epsilon}{\partial x_2}\right) - 1.43\frac{\epsilon}{k}\overline{u_1'u_2'}\frac{\partial U_1}{\partial x_2} - \frac{\epsilon}{k}\left[\frac{4.44}{3}k\frac{\partial U_1}{\partial x_1} + 1.92\epsilon\right] - \underline{\frac{2\rho_p}{\rho}\frac{\epsilon}{t_*}} \tag{11}$$

This form of the (k-ϵ) model has been developed for gas-particle turbulent flows by Chen and Wood (1984b). It is based on the (k-ϵ) model of Hanjalic and Launder (1980) but has been modified to account for the presence of the particles. The underlined terms are added sink terms, which are due to the added velocity gradients created by the particles. The added term in the k equation is valid for any size particle, but the additional term in the ϵ equation is only valid when $t_* \gg \tau$, where $\tau = (\nu/\epsilon)^{1/2}$ is the time scale of the dissipative eddies.

The diffusivities ν_p and D_t are derived from the fluid phase turbulent viscosity

$$\nu_p = \frac{\nu_t}{(1 + t_*/t_e)} \tag{12}$$

Here t_e is the time scale of the energetic turbulent eddies given by $t_e = 0.165\, k/\epsilon$. Finally $D_t = \nu_t/Sc_t$ where the turbulent Schmidt number $Sc_t = 0.7$, based on turbulent mass transfer data.

RESULTS AND DISCUSSION

Equations 2–12 are solved simultaneously subject to appropriate boundary conditions and initial conditions. A novel numerical scheme for solving these equations was developed (Chen and Wood, 1984a). In this scheme initial density and velocity profiles are marched downstream using the centered difference scheme of Keller (1970). These initial profiles are obtained by computing the fluid field profiles for fully developed pipe flow. It was also assumed that the mean axial velocity profiles of the particles was equal to that of the fluid at the jet exit. The radial mean velocities of the two phases were taken to be equal throughout the calculation (Melville and Bray, 1979). The particulate phase mean velocity was taken equal to that of the fluid at the jet exit and its mean density profile was uniform. All of these conditions are in agreement with the experimental data of Girshovich et al. (1982) and Moderass et al. (1984).

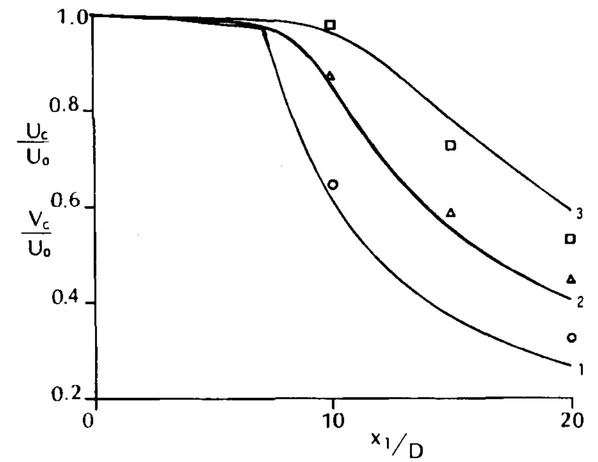

Figure 1. Centerline velocity decay. Curve 1 ○, clean jet, U_c/U_o. Curve 2 △, fluid phase of two-phase jet, U_c/U_o. Curve 3 □, particulate phase, V_c/U_o. Dotted curve, model of Chen and Wood (1984b), data of Moderrass et al. (1984); d_p = 50 µm, ρ_p/ρ = 0.32.

It is well known that the presence of the dispersed phase causes the jet to be more coherent. That is, the centerline velocity decay is reduced with a corresponding reduction in the spreading rate. These reductions increase with increasing particle size and/or loading. In Chen and Wood (1984b) it was shown that the spreading rate is well predicted using this type of turbulence closure modeling. All of the cases calculated in that paper (which were in excellent agreement with the data of Wall et al., 1982) were recomputed using the present model and similarly good predictions of the spreading rate were obtained. However, the mean centerline velocity decay is affected by allowing slip to exist between the phases. This is shown in Figure 1. Curve 1 is the clean jet, curve 2 the gas phase velocity of the two-phase jet, and curve 3 the particulate phase velocity. Notice that from the end of the potential core on, because of the particles' inertia, the particulate phase velocity is higher than the gas velocity. The dotted curve was obtained using the model of Chen and Wood (1984b), and since no mean slip was allowed it represents the velocity of both phases.

In Figure 2 detailed mean field profiles at the downstream location $x_1 = 14R$ are plotted. The density of the particulate phase is well predicted, but the very large slip between the phases observed by Girshovich et al. (1982) near the edge of the jet is not. This discrepancy may be a consequence of using Stokes law to compute the interaction force between the particles and gas. For Stokes law to apply, the particle Reynolds number based on the local slip velocity should be small. However at this downstream location $Re_p \approx 20$ on the centerline. A drag coefficient that depends on Re_p could be implemented, but because of the added nonlinearity this was not attempted. It should be pointed out that the largest error occurs at the jet edge where there are very few particles as shown by the mean density profile in Figure 2. Hence the net effect of this error on the mean density profile is minimal. It should also be noted that the mean density profile is narrower than the mean gas velocity profile despite the fact that $Sc_t < 1$.

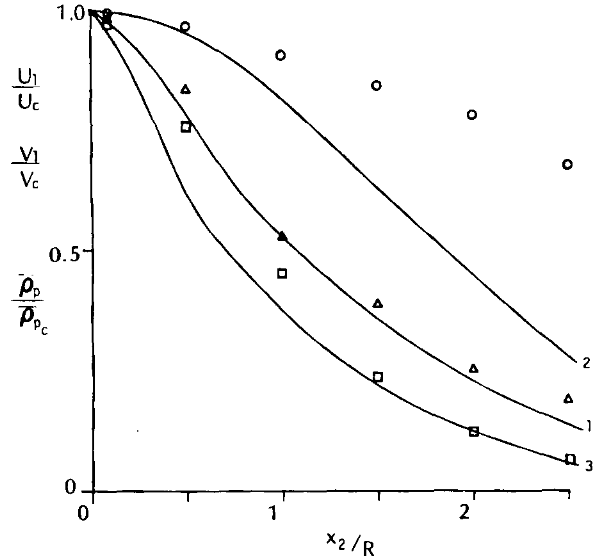

Figure 2. Mean profiles. Curve 1 △, mean fluid phase velocity, U_1/U_c. Curve 2 O, mean particulate phase velocity, V_1/V_c. Curve 3 □, mean particulate phase density ρ_p/ρ_{p_c}. Data of Girshovich et al (1982); d_p = 45 μm, ρ_p/ρ = 1.0, x_1/R = 14.

This apparent anomaly is due to the higher velocity of the particulate phase through a control volume, which compensates for its more rapid diffusion.

The effect of the particles on the gas phase turbulence structure is shown in Figures 3 and 4. In Figure 3 the turbulence kinetic energy is plotted against the normalized cross-stream distance. The calculations are compared with the data of Moderrass et al. (1984) taken at $x_1 = 20D$. Also shown for comparison is the clean jet profile. It has been previously shown (Chen and Wood, 1984b) that the $(k$-$\epsilon)$ model does only a fair job of predicting the kinetic energy in this region of the jet but the relative decrease in turbulence energy caused by the presence of the particles is well predicted. That is, for a loading $\rho_p/\rho = 0.32$, k is reduced by 40% as predicted by the model. In Figure 4 similar behavior for the mean Reynolds shear stress is shown.

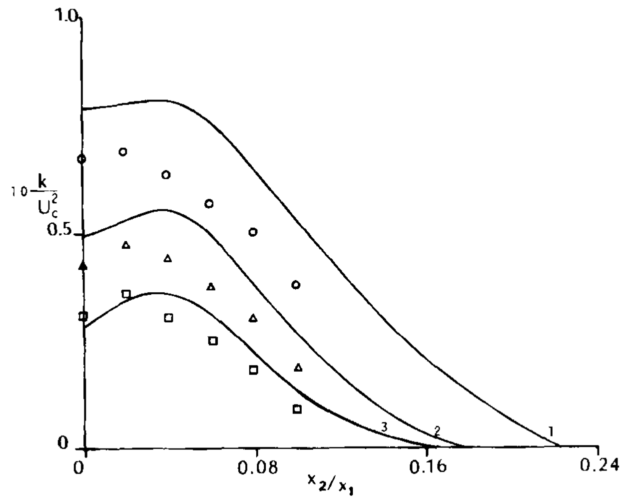

Figure 3. Turbulence kinetic energy. Curve 1 O, clean jet. Curve 2 △, ρ_p/ρ = 0.32. Curve 3 □, ρ_p/ρ = 0.85. Data of Moderrass et al. (1984), d_p = 50 μm, x_1/D = 20.

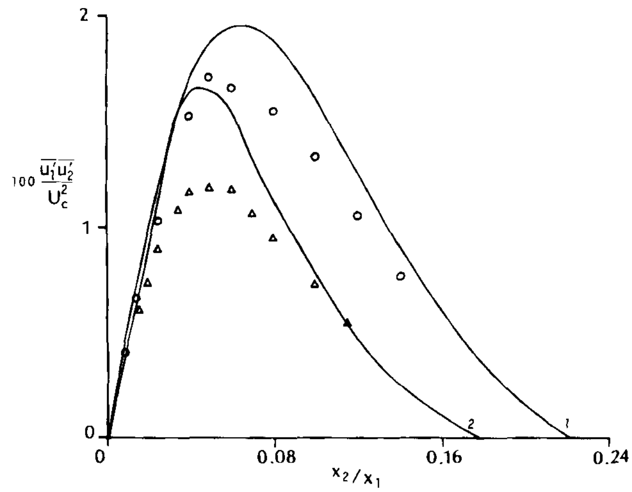

Figure 4. Mean Reynolds shear stress. Curves have same meaning as Figure 3.

In summary, reasonable predictions of the slip between the phases can be obtained by solving momentum equations for each phase coupled by a Stokes drag term. Improvements could be made by using a more accurate drag coefficient but at the expense of added computational cost. Important mean field characteristics such as the entrainment rate, the spreading rate, and the gas velocity profile are well predicted even if it is assumed that no mean slip exists between the two phases (cf. Chen and Wood, 1984b). Hence a model such as that proposed here need only be used if a heat or mass transfer coefficient must be calculated simultaneously.

NOTATION

d_p	= particle diameter
D	= pipe diameter
D_t	= particle diffusivity
F_{pi}	= particle/fluid interaction term, Eq. 1
k	= turbulent kinetic energy per unit mass
p,P	= pressure, mean pressure
R	= initial jet radius
Re_p	= particle Reynolds number = $(d_p\|V-U\|)/\nu$
Sc_t	= turbulent Schmidt number
t_e	= time scale characteristic of energy containing eddies
t_*	= characteristic response time of the particles
u_i,u_i',U_i	= fluid velocity, fluctuating velocity, mean velocity
v_i,v_i',V_i	= particulate phase velocity, fluctuating value, mean value
x_1,x_2	= axial coordinate, transverse coordinate

Greek Letters

ϵ	= dissipation rate per unit mass
Λ	= integral length scale of turbulent eddies
μ,v	= fluid viscosity, kinematic viscosity
v_p	= turbulent viscosity of particulate phase
v_t	= turbulent viscosity of fluid phase
ϕ	= particulate phase volume fraction
ρ	= fluid density
ρ_p	= mean particulate phase density
ρ_s	= solid material density
τ	= Kolmogorov time scale

Subscripts

i,j	= coordinate directions, 1 = streamwise, 2 = transverse
0	= initial value at beginning of jet
c	= centerline value
f	= turbulent fluid phase
p	= particulate phase
s	= solid phase

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